An experiment on the stability of hypersonic laminar boundary layers

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An experimental investigation of the hydrodynamic stability of the laminar hypersonic boundary layer was carried out with the aid of a hot-wire anemometer. The case investigated was that of a flat surface at zero angle of attack and no heat transfer.

The streamwise amplitude variation of both natural disturbances and of disturbances artificially excited with a siren mechanism was studied. In both cases it was found that such small fluctuations amplify for certain ranges of frequency and Reynolds number R_θ, and damp for others. The demarcation boundaries for the amplification (instability) zone were found to resemble the corresponding limits of boundary-layer instability at lower speeds. A 'line of maximum amplification' of disturbances was also found. The amplification rates and hence the degree of selectivity of the hypersonic layer were found, however, to be considerably lower than those at the lower speeds. The disturbances selected by the layer for maximum amplification have a wavelength which was estimated to be about twenty times the boundary-layer thickness δ.

I. Introduction

The problem of transition of a fluid flow to turbulence has for a long time attracted the attention and toil of both theoreticians and experimentalists. At hypersonic speeds the problem of transition is one of pressing importance, since the differences in skin friction and heat transfer between laminar and turbulent flow are large enough to alter radically the performance of, say, a hypersonic aircraft or a long-range missile.

Of the various hypotheses put forth to describe the mechanism of transition, one which has been steadily gaining ground is the small-disturbance hypothesis. It states that within some range of the mean-flow parameters, certain infinitesimal disturbances found in the laminar part of the flow (whether wake, jet, or boundary layer) amplify as they progress downstream and eventually break up into the random pattern that we identify as turbulence. The small-disturbance theory itself does not deal with the problem of turbulence; it merely examines whether and how disturbances amplify and damp in the laminar flow, and is therefore a theory of hydrodynamic stability.

The stability of parallel flows was treated by Rayleigh (1880), Prandtl & Tietjens (1925), Heisenberg (1924), and Tollmien & Schlichting (1930). Recently the stability of the boundary layer of a compressible fluid with heat transfer was investigated by Lees & Lin (1946), Lees (1947), and Dunn & Lin (1955). The small-disturbance theory of boundary-layer flows enjoyed its first success with the classic experiments of Schubauer & Skramstad (1948), and of Liepmann (1943), which showed the existence of the laminar waves of Tollmien and Schlichting and which verified, both qualitatively and quantitatively, many of the prominent features of the theory. Recently, Laufer & Vrebalovich (1956), in studying the flow over a flat plate at a Mach number of about 2, obtained data similar to that obtained by Schubauer & Skramstad at low speeds.

At hypersonic speeds the large temperature differences existing across the boundary layer bring to the foreground some hitherto unsettled points in the analysis of the stability problem.* The present experiment (Demetriades 1958) was undertaken in order to provide an understanding of the stability of the hypersonic layer, and, more specifically, to see whether there exist ranges of the defining parameters where small disturbances damp or amplify, to obtain information on the bounds of these ranges, and to study the manner of amplification or damping. For this purpose the experiment was limited to the simple case of a smooth flat plate at zero heat transfer and zero pressure gradient.

2. Technique and equipment

The downstream progress of a small disturbance

$$Q' = q(y)\, e^{i\alpha(x-ct)}$$

in a laminar boundary layer will be arrested or enhanced depending on whether c_i, the imaginary part of the propagation velocity c, is negative or positive. When c_i is zero the disturbance is neutral and, for a fixed Mach number, the small-disturbance theory produces a unique relationship between wave-number α and Reynolds number R_θ, based on momentum thickness. This relationship divides the (α, R_θ) plane into a stable and an unstable region.

Consideration of the (α, R_θ) plane suggests a suitable experimental technique. If one measures the root-mean-square amplitude of a particular disturbance at successive streamwise positions in the boundary layer, one should be able to identify the extrema in this amplitude variation with points falling on the dividing curve. In fact, the primary aim of this investigation was to see whether these extrema defined a unique locus at a hypersonic Mach number.

In addition to studying the streamwise history of a disturbance of a given frequency, a second obvious method is to study the energy spectrum of disturbances at a preselected, fixed point in the layer. This approach is particularly tempting, since the small-disturbance theory in this case predicts energy concentrations in certain portions of the spectrum and since these peaks have been easy to observe at the lower flow velocities (Schubauer & Skramstad 1948; Liepmann 1943). In order to apply these two methods, an instrument capable of temporal and spatial measurements on a minute scale is required. Such

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* A theoretical study of the stability of the hypersonic laminar boundary layer has been undertaken by E. Reshotko and L. Lees at the Guggenheim Aeronautical Laboratory, California Institute of Technology.

an instrument is the hot-wire anemometer. The applicability and reliability of this instrument in high-speed flows has been demonstrated by many workers, notable Kovasznay (1954), Morkovin (1956), and Laufer & McClellan (1955). Calibration of the anemometer or resolution of its output into components due to velocity, density or temperature fluctuations is unnecessary here since the stability boundaries remain unaltered when different types of disturbances are considered.

The present investigation was carried out in Leg 1 of the GALCIT hypersonic wind tunnel. Two different flat-plate models were used. The survey plate used to study the stability of natural fluctuations was 26 in. long, had a sharp leading edge and smooth finish, and was aligned within the test rhombus at zero incidence (cf. figure 1). The siren plate was almost identical to the survey plate externally but contained a mechanism for producing fluctuations of controllable amplitude and frequency. A 4 in. diameter thin brass siren disk imbedded under the surface had sixty regularly spaced holes drilled along its circumference, while another seven small slits 0·060 × 0·028 in. were drilled along a straight line 1·75 in. downstream from the leading edge, in a cover-plate emplaced over the disk. The seven slits were spaced so that they were either in complete coincidence or in complete anti-coincidence with the holes in the siren disk as the latter rotated underneath; an air pulse was therefore injected into the flat-plate boundary layer for each 6° of disk rotation. By regulating the speed of a motor driving the disk, frequencies from about 1 up to about 40 kc/s were attainable.

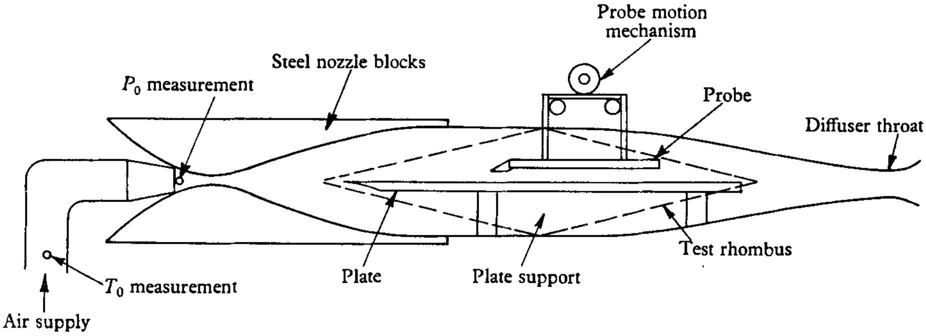


FIGURE 1. Schematic diagram of GALCIT Leg 1 hypersonic tunnel test section showing flat plate and probe installation.

The hot-wire anemometer probe is pictured in figure 1. The probe can be moved vertically above the plate surface, as well as in a stream-wise direction, providing the hot wire with two degrees of freedom. The hot-wire element was soft-soldered on the tips of two heat-treated sewing needles which were, in turn, cemented in grooves on the detachable probe head. The probe head is 'plugged in' to a socket in the front of the probe strut through which pass a pair of co-axial cables, with very small diameter, maintaining electrical connexion with the electronic apparatus. The wire was operated at constant current drawn from five 45 V cells in series. The instrumentation included a pre-amplifier with a maximum gain of about 3200, a compensating amplifier to correct for the thermal lag of the wire, and band-pass filtering apparatus. The magnitude of the r.m.s.

voltage was measured with a vacuum thermocouple in series with a 92 Ω microammeter. A display-type wave analyser was also found to be very useful in this work. Energy spectra were recorded with a harmonic wave analyser and a thermocouple-microammeter combination.

Platinum-rhodium wires of 0·0001 in. diameter measuring from 0·02 to 0·03 in. in length were used exclusively in this work. The wires were always held normal to the flow direction and parallel to the plate surface, so that they were sensitive to the u' (streamwise) perturbation velocity component. It was found expedient to anneal and prestretch the wire in the hypersonic stream itself; thus treated some wires were used for several tens of hours without any appreciable changes in electrical characteristics or any visible deterioration. No severe strain-gauge effects (Morkovin 1956) were encountered. Table 1 presents some of the pertinent operating conditions of the hot wire.

Material	Platinum 90% Rhodium 10%
Diameter	0·0001 in.
Length	0·02–0·03 in.
Aspect ratio	200–300
Mach number	5·8
Wire Reynolds number Re_d (free-stream conditions)	Order of 1
Wire Nusselt number Nu_d (free-stream conditions)	Order of 0·1
Resistance (equilibrium) R_e	30–40 Ω
Total pressure	28–75 p.s.i.g.
Total temperature	225 °F
Dynamic loading	0·8–1·6 p.s.i.
Error in sensitivity coefficients due to 'end effects'	5–10%
Time constant	$0{\cdot}2\text{–}1 \times 10^{-3}$ sec
Maximum current through wire	About 10 mA

TABLE 1. Typical operating conditions for the hot-wire anemometer.

The laminar boundary layer over the survey flat-plate at zero incidence at $M = 5{\cdot}8$ was studied in some detail by Korkegi (1954) and also by McMahon (1958) and the author. Laminar boundary-layer profiles obtained by using a total-pressure tube yielded the layer thickness δ and the momentum thickness θ along the centreline of the plate. A series of such profiles taken along the centreline of the survey plate disclosed that boundary layer similarity is definitely preserved. The boundary-layer growth over the siren plate was also measured with flow conditions simulating those under which the presently described experiment was carried out. The agreement between these measurements and the measurements in the boundary layer on the survey plate was good. The range of R_θ varied between 350 and 2050.

The free-stream turbulence was also investigated in the region of interest, namely, in the region of the test section which would be normally occupied by the flat-plate boundary layer. The absolute level of the turbulence was measured at a distance of 17·5 in. from the nozzle throat. The data yielded values of the order of 0·4 % in both the mass flux and the temperature fluctuation in the range 0–100 kc/s, with little variation due to the tunnel supply pressure. Thus the free-stream disturbance energy contained in a narrow frequency band (say, between 20 and 25 kc/s) was one, or possibly two, orders of magnitude smaller than the allowable level for purposes of this experiment.

3. Measurements of the natural fluctuations

The growth or decay of fluctuations which exist naturally in the laminar boundary layer was measured on the survey plate by moving the hot wire along the layer at chosen constant values of y/δ. The hot-wire output at a series of selected frequencies was recorded with the aid of the harmonic wave analyser and the vacuum thermocouple-microammeter combination. Keeping y/δ constant as the hot wire was moved streamwise in increments of x was relatively simple, since the wire resistance R_w is a function only of y/δ. As the probe was moved downstream by steps it was also raised slightly each time, so that the resistance-measuring galvanometer was again balanced at the previously set value of R_w. Some typical data, taken at values of y/δ ranging from 0·7 to 0·9, are singled out and shown in figure 2. The abcissa of these graphs is the distance of the wire from the leading edge of the plate; the ordinate is the micro-ammeter reading, which is proportional to the mean-square voltage fluctuation $\overline{(\Delta e')^2}$ and thence to the square of the fluctuation amplitudes, that is to the fluctuation energy. Figure 2(*a*) shows the appearance of a maximum at a frequency of 45 kc/s, and figure 2(*b*) a sequence of such maxima. The x-values at which these maxima occurred, when used in conjunction with the tunnel-flow parameters and the local boundary-layer momentum thickness, gave points in the plot of the non-dimensional frequency $\beta_r \nu_\infty / U_\infty^2$ versus the Reynolds number R_θ; here β_r is proportional to the disturbance frequency, and ν_∞ and U_∞ the kinematic viscosity and fluid velocity, respectively, in the free stream.

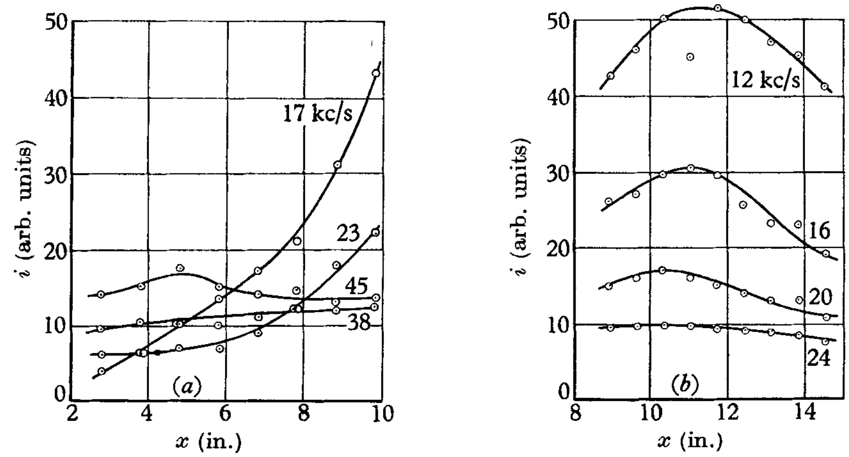
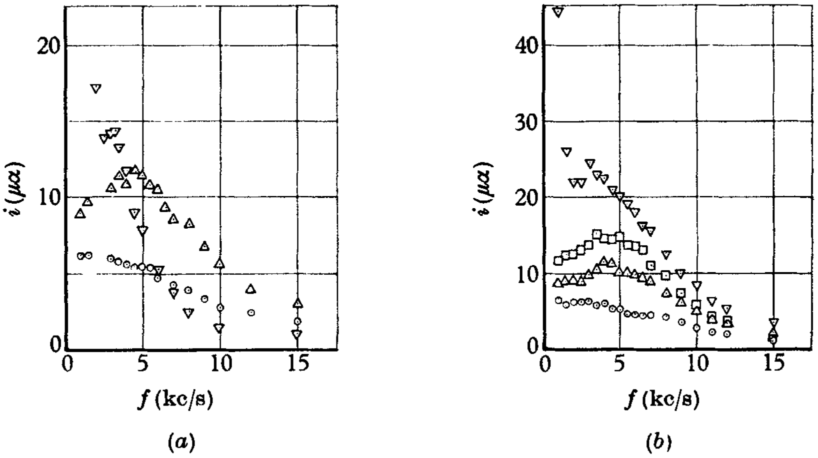


FIGURE 2. Typical variation of hot-wire output intensity due to natural fluctuations. (*a*) P_0 = 35 p.s.i.g.; (*b*) P_0 = 45 p.s.i.g.

These points fell, within some scatter, on a line which should thus be the upper neutral branch of the boundary-layer stability diagram; this line will not be presented here, however, since it generally agreed with the branch found by means of the more reliable artificial-fluctuation method. Since no consistent appearance of minima was observed, the lower neutral branch was not discernible for the natural fluctuations.

As pointed out earlier, the amplitude spectrum of the natural fluctuations should exhibit a maximum when measured at any streamwise location in the boundary layer. In searching for this property in the hypersonic boundary layer the survey flat plate was again employed, and the wire-output spectrum was recorded for various combinations R_θ and y/δ. After prolonged measurements at various ranges of output frequency and Reynolds number a series of energy peaks were discovered in the intervals from 0 to 10 kc/s in frequency and 800–2000 in R_θ. The data are shown in figure 3.



(*a*) (*b*)

FIGURE 3. Hot-wire mean-square output spectra in the laminar boundary layer, $M = 5{\cdot}8$, $T_0 = 225\,^\circ$F. (*a*) $P_0 = 28$ p.s.i.g.; ⊙, $x = 9{\cdot}2$ in.; △, $x = 11{\cdot}8$ in.; ▽, $x = 16{\cdot}5$ in. (*b*) $P_0 = 35$ p.s.i.g.; ⊙, $x = 9{\cdot}2$ in.; △, $x = 11{\cdot}5$ in.; □, $x = 14{\cdot}0$ in.; ▽, $x = 16{\cdot}5$ in.

By far the most interesting feature of the energy spectra of figure 3 is the absence of any completely predominant energy peak corresponding to the data for incompressible flow obtained by Schubauer & Skramstad (1948). The energy peak is barely discernible for $R_\theta = 880$, although this lower limit is presumably strongly dependent on the free-stream turbulence level and other characteristics of the experimental environment. For larger values of x (or R_θ) the relative magnitude of the peak increases, but certainly not in a manner permitting it, for example, to be viewed on an oscilloscope without band-pass filtering. At the largest x-values attainable with the hot-wire probe traverse the peak is finally obscured by an abrupt increase of the energy contained from 0 to 2 kc/s in the spectrum. Figure 4 presents the points defined by the frequency-at-maximum-amplitude of curves such as those given in figure 3. The two solid curves on figure 4 are the neutral branches discussed in § 4.

A final point of interest concerns the value of y/δ at which the measurements just described were carried out. More specifically, one is interested in finding out how the amplitude peak, at the fluctuation frequency that the boundary layer has selected for maximum amplification at some R_θ, changes with y/δ. For this purpose the wire was held at some constant R_θ and was traversed across the boundary layer from the solid surface toward the free stream in steps of about $\frac{1}{5}\delta$ each; at each step the energy spectrum was recorded. Very near the plate there is no peak in the spectrum. As the wire is raised through the layer the peak, in this case at about 2·8 kc/s, appears, becomes most prominent for values of y/δ between 0·6 and 1, persists briefly in the free stream and finally disappears for $y/\delta = 1{\cdot}34$.

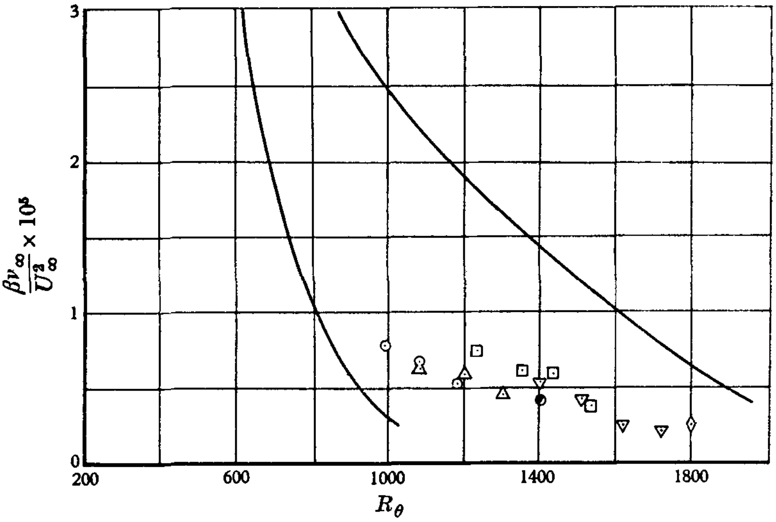


FIGURE 4. Points defining the 'line of maximum amplification'. ⊙, $P_0 = 28$ p.s.i.g.; △, $P_0 = 35$ p.s.i.g.; ◐, $P_0 = 40$ p.s.i.g.; ⊡, $P_0 = 45$ p.s.i.g.; ▽, $P_0 = 55$ p.s.i.g.; ◇, $P_0 = 60$ p.s.ig.

At the conclusion of the work with the natural fluctuations encouraging evidence had been accumulated to the effect that the laminar hypersonic boundary layer observed, at least in a general way, the stability rules of the small-disturbance theory. A demarcation boundary between an unstable and a stable region had been found and characteristic fluctuations at selected frequencies had also been found and their dependence on R_θ ascertained (figure 4). The amplification rates that had been found, however, were quite low and it was suspected that natural causes such as the free-stream turbulence might have been concealing the detailed features of the stability diagram, particularly the lower neutral branch. For this reason the study of disturbances injected with the 'siren' mechanism described in the next section was undertaken.

4. Measurement of the artificial fluctuations

In utilizing the siren plate for the measurement of artificial fluctuations, care was taken to see that the amount of air injected did not cause premature transition. For very small injection rates it was almost impossible to detect the induced fluctuation downstream of the slits. As the injection rate increased the energy peak corresponding to the siren frequency appeared and increased in magnitude. When some critical injection rate was reached the peak magnitude increased no further, but an abrupt large increase of the wire output was noted over the entire frequency range; raising the injection rate further had no significant effect on the mean-square level of the wire output. This critical injection rate was, therefore, the one at which the layer became turbulent at the location of the wire. Knowledge of this rate enabled one to choose an injection rate appropriate for taking data and thus to estimate, even though roughly, the comparative magnitude of the fluctuation amplitudes. Generally, data were taken at all subcritical rates, the guiding consideration being the appearance and clarity of the energy peak at the siren frequency. All artificial fluctuation data were taken by using the band-pass filter to suppress signals at all frequencies below and above that of the siren.

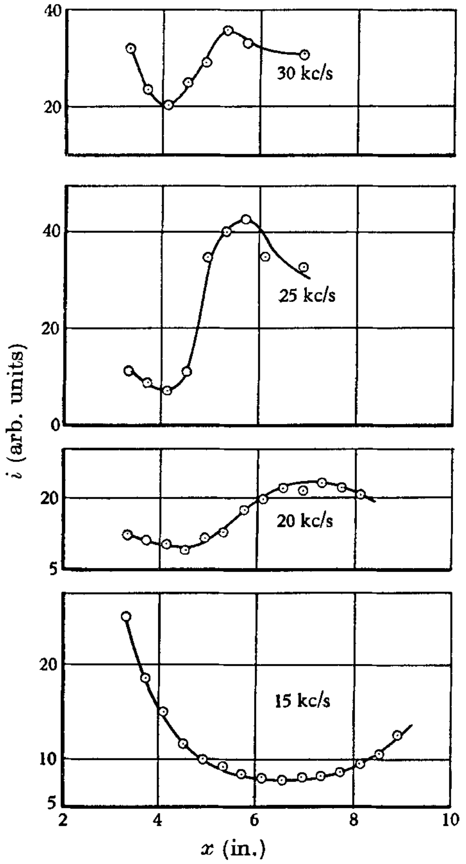


FIGURE 5. Typical hot-wire output variations due to artificial fluctuations. $P_0 = 30$ p.s.i.g.;

The results of the constant-frequency surveys appear in figures 5 and 6. The value of y/δ was maintained constant for each survey using the method described in the previous section. Figure 5 shows typical surveys at constant frequency exhibiting extrema in the amplitude variation. In almost all cases the variation of the location of the extrema (maxima or minima) with fluctuation frequency is apparent. As a rule the signals were very weak for y/δ much smaller than unity and disappeared rapidly once the probe was moved into the free stream.

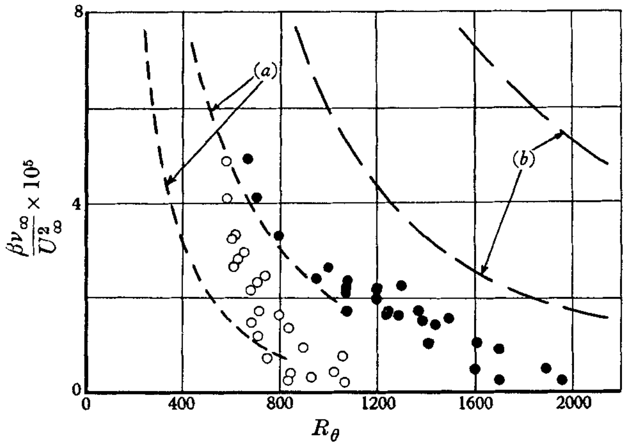


FIGURE 6. Experimentally determined upper neutral branch (solid circles) and lower branch (open circles) at $M = 5{\cdot}8$. (*a*) $M = 2{\cdot}16$ (Laufer & Vrebalovitch; (*b*) $M \simeq 0$ (Schubauer & Skramstad).

The results of the artificial-fluctuation study are plotted in figure 6 in the usual non-dimensional frequency plot, where they are also compared with similar findings at lower Mach numbers. It is apparent that the general shape of the region of instability is preserved throughout the range of Mach numbers from 0 to 5·8. Another consistent trend appears to be the gradual decrease, with increasing Mach number, of the area enclosed by the neutral stability boundaries, at least above a certain low non-dimensional frequency. The scatter of the data in figure 6 is of undetermined origin. No systematic effect of the supply pressure P_0 on the scatter was observed, nor did there seem to be an effect of y/δ and hence also of the wire sensitivity. The effect of the air injection rate, on the other hand, is difficult to estimate properly since it may be significant in two independent ways, namely, by affecting the fluctuation amplitude and also by changing the boundary-layer thickness. Both these effects are thought to be small.

By far the most remarkable feature of the work with the artificial fluctuations was that it made possible the discovery of the lower neutral branch. It is thought that the inability to discover the lower branch for the natural fluctuations lies mainly in the interference caused by the free-stream turbulence level. In order to substantiate this conjecture, a comparison was made of the hot-wire signal in the boundary layer a few inches downstream of the leading edge of the survey plate, and the signal obtained at the same geometrical position but with the plate removed from the wind tunnel: these two signals were found to be of about the same magnitude, both quite smaller than signals obtained with the siren mechanism operating. In fact, an insight into all the differences between the natural-fluctuation and the artificial-fluctuation data could be gained by considering the role of the free-stream turbulence.

5. The critical layer

One of the features of the theory of boundary-layer stability is that the phase velocity c_r of the laminar fluctuations is such that, for fluctuations which are 'subsonic' relative to the free stream,

$$1-(1/M_\infty) < (c_r/U_\infty) < 1,$$

so that for $M_\infty = 5{\cdot}8$ we have

$$0{\cdot}828 < (c_r/U_\infty) < 1.$$

The point in the boundary layer where c_r is equal to the local air velocity is called the critical point and the surface locus of these points over the flat plate is called the critical layer. In the present instance it was assumed that

$$c_r = 0{\cdot}9U_\infty = \text{constant},$$

which implies a maximum probable error of less than 10 %. Using this approximation to c_r, we can now compute the fluctuation wavelengths to the same degree of accuracy with the aid of the relation

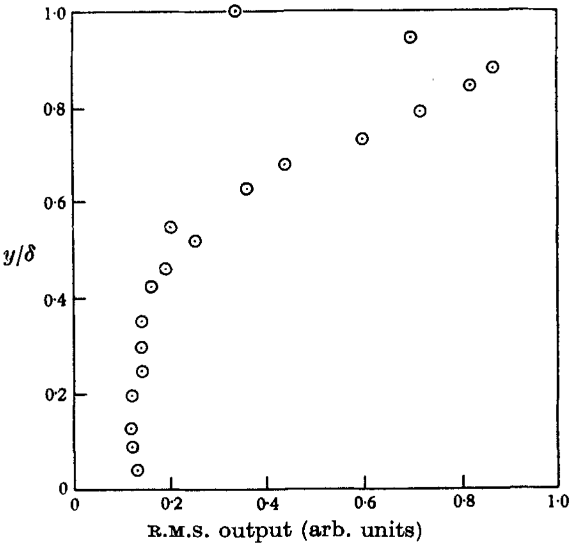


FIGURE 7. Variation, across the laminar boundary layer, of the hot-wire output integrated over all frequencies. $R_\theta \approx 1400$.

$$\alpha_\theta = \frac{2\pi\theta}{\lambda} = \frac{R_\theta}{c_r/U_\infty}\left(\frac{\beta_r \nu_\infty}{U_\infty^2}\right) = 1{\cdot}11R_\theta\left(\frac{\beta_r \nu_\infty}{U_\infty^2}\right),$$

where θ is the momentum thickness of the boundary layer. The result is shown on figure 8, with the theoretical computations for $M = 0$ and $M = 1{\cdot}3$ from Lees (1947) also shown for comparison. With the aid of figure 4 we also observe that the fluctuations receiving maximum amplification in the range from $R_\theta = 1000$ to 2000 have an α_θ of about 0·01. Since there is a difference of a factor of about 30 between θ and the boundary-layer thickness δ, the corresponding wavelengths are of the order of 20δ, or from 3 to 6 in.

The location of the critical layer was also discovered independently. Keeping the current constant, the hot wire was traversed across the boundary layer and the change in the root-mean-square output, integrated over all frequencies was recorded. It was immediately noticed that the output increased rapidly as the wire approached the edge of the layer, came to a maximum at $y/\delta \doteqdot 0{\cdot}9$ and then decreased to some constant value in the free stream, as shown on figure 7. This energy concentration could not be explained on the basis of any assumed structural vibrations of the probe support or the plate model itself, and it was concluded that it was due to the cumulative effect of the maximum in the amplitude distribution factor $q(y)$ of the various fluctuations, and therefore that the value of y/δ at which this prominence appeared gave the position of the critical layer.* From Korkegi (1954) we can further deduce that at $y/\delta = 0{\cdot}9$ the local flow velocity is about $0{\cdot}98U_\infty$, which is within the expected 10 % error in the assumption $c_r = 0{\cdot}9U_\infty$.

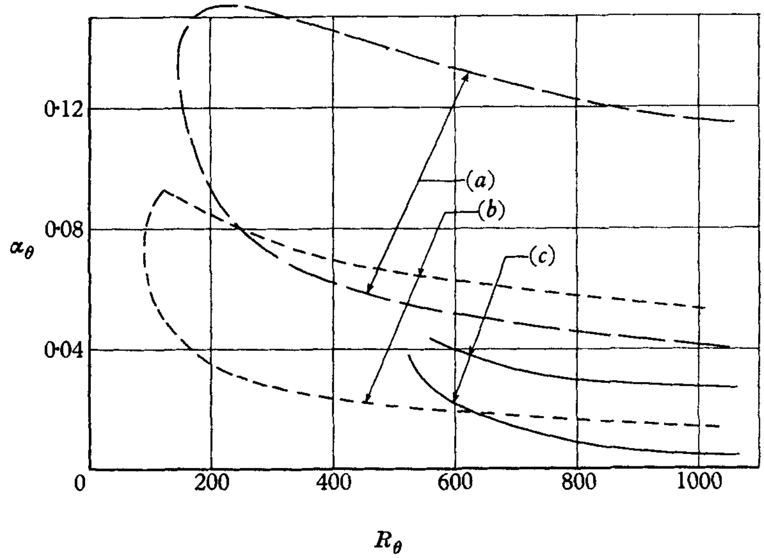


FIGURE 8. Wave-number α_θ *vs.* R_θ for neutral stability of the boundary layer. (*a*) $M \simeq 0$ (Blasius flow); (*b*) $M = 1{\cdot}3$ (Lees); (*c*) $M = 5{\cdot}8$ (present data).

6. Conclusions

The laminar boundary layer growing over a flat plate at zero incidence in a hypersonic air stream has been investigated with the aid of a hot-wire anemometer. The main purpose of the investigation has been to see whether the hypersonic layer is hydrodynamically stable or unstable to small disturbances in the flow variables. The following results have been obtained:

1. For certain ranges of the flow conditions and the frequency of the small fluctuations the boundary layer is stable, while for all other ranges of the same parameters it is unstable, in a manner resembling the theoretically and experimentally ascertained stability of boundary-layer flows at lower speeds.

* Recent computations by E. Reshotko at this laboratory have shown that, at Mach numbers of the order of 5, a strong concentration of the temperature fluctuation amplitude $q(y)$ occurs at the critical layer. Since the hot wire is most sensitive to the temperature fluctuation at the edge of the layer, it is inferred that the peak of figure 7 is due mostly to temperature fluctuations and thus marks the location of the critical layer.

2. The amplication rates observed are much lower than those observed at low speeds. Similarly, the characteristic fluctuations selected by the laminar layer for maximum amplification have a broad band width and hence are more difficult to detect than those at the lower speeds.
3. The wavelengths of these characteristic fluctuations are at least one order of magnitude larger than the local boundary-layer thickness δ.

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